Midterm 1 Review

Domains

f - function

Domain of f = Those x for which $f(x)$ makes sense.

Things to look out for :

1/ Denominators being 0

2/ Negative numbers inside square roots, ln.

Example : $f(x) = \sqrt{\dfrac{x+1}{x+2}}$

$\dfrac{x+1}{x+2} \geq 0 \implies x+1 \geq 0,\ x+2 > 0 \implies x \geq -1,\ x > -2$

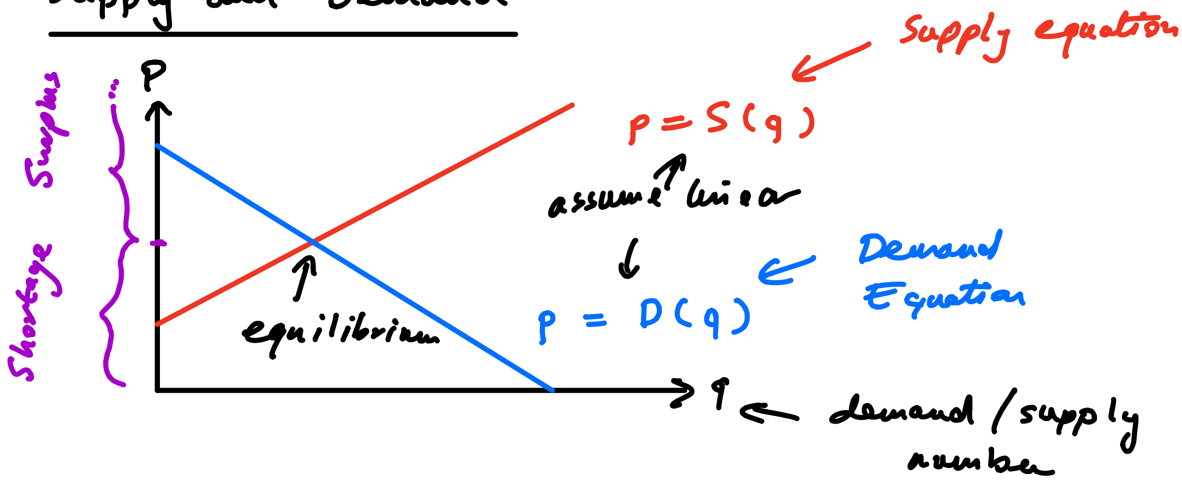
or

$x+1 \leq 0,\ x+2 < 0 \implies x \leq -1,\ x < -2$

$\implies \quad x \geq -1$ or $x < -2$

$\implies$ Domain is $(-\infty, -2) \cup [-1, \infty)$

Supply and Demand



Cost, Revenue, Profit

$C(x)$ = cost of producing x units
$R(x)$ = Revenue from selling x units
$P(x) = R(x) - C(x)$ = profit from producing and selling x-units

$R(x) = C(x) \Rightarrow$ Break even
$R(x) > C(x) \Rightarrow$ Profit
$R(x) < C(x) \Rightarrow$ Loss
$C'(x), R'(x), P'(x)$ = marginal cost, revenue, profit

Compound Interest

Compounding m times per year :

$$F(t) = P\left(1+\frac{r}{m}\right)^{mt}$$

annual interest rate $(0 \le r \le 1)$

Balance at time t

Initial deposit

time in years (whole number)

Continuous Compounding :

$$F(t) = Pe^{rt}$$

t in years (any number)

important constant $e = 2.71828\ldots$

Limits

$\lim_{x \to a^+ / a^- / a} f(x) = L \iff$ $f(x)$ approaches L as x approaches (but does not equal) a from above / below / both sides

$\lim_{x \to \infty / -\infty} f(x) = L \iff$ $f(x)$ approaches L as x grows positively / negatively without bound.

Core Examples :

f continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$.

Polynomials, Rational Functions, Exponential Functions, Logarithmic Functions, power functions are continuous everywhere in their domain.

Same is true of sums, products, quotients, compositions of above.

Example 1 $\lim_{x \to 3} e^{x^2+2} = e^{3^2+2} = e^{11}$.

2/ $f(x) = \begin{cases} e^x & \text{if } x \geq 1 \\ x^2 - 3 & \text{if } x < 1 \end{cases}$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} e^x = e^1 = e$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 - 3 = 1^2 - 3 = -2$$

$\Rightarrow \lim_{x \to 1} f(x)$ DNE.

<u>Limit Laws</u> : (Assuming all of following limits exist)
(Valid for all types of limit)

<u>Sum Law</u> : $\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

<u>Product Law</u> : $\lim_{x \to a} (f(x) g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

<u>Exponential Law</u> : $\lim_{x \to a} b^{f(x)} = b^{\left(\lim_{x \to a} f(x)\right)}$

<u>Log. Law</u> : $\lim_{x \to a} \log_b (f(x)) = \log_b \left(\lim_{x \to a} f(x) \right)$

<u>Power Law</u> : $\lim_{x \to a} (f(x))^r = \left(\lim_{x \to a} f(x) \right)^r$

<u>Quotient Law</u> : If $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = \begin{cases} \frac{L}{M} & \text{if } M \neq 0 \\ \text{DNE} & \text{if } L \neq 0,\ M = 0 \\ \text{UNCLEAR} & \text{if } L = M = 0 \end{cases}$$

If case 2 we can go further :

$$L > 0,\ M = 0^+ \Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = \infty$$

approaches 0 positively

approaches 0 negatively

$$L > 0,\ M = 0^- \Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = -\infty$$

$$L < 0,\ M = 0^+ \Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = -\infty$$

$$L < 0,\ M = 0^- \Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = \infty$$

__Example 1/__ $\displaystyle \lim_{x \to -1} \frac{x+2}{(x+1)(x-3)} = ?$

$$\lim_{x \to -1} x+2 = -1+2 = 1 > 0$$

-1

$x+1 < 0$ $\quad$ $x+1 > 0$

$x-3 < 0$ $\quad$ $x-3 < 0$

$$\Rightarrow \lim_{x \to -1^+} (x+1)(x-3) = 0^-$$

$$\lim_{x \to -1^-} (x+1)(x-3) = 0^+$$

$$\Rightarrow \lim_{x \to -1^+} \frac{x+2}{(x+1)(x-3)} = -\infty$$

$$\lim_{x \to -1^-} \frac{x+2}{(x+1)(x-3)} = \infty$$

$\Rightarrow \displaystyle \lim_{x \to -1} \frac{x+2}{(x+1)(x-3)}$ DNE and is neither $\pm\infty$.

2/ $$\lim_{x \to -\infty} \frac{6x^2+1}{7x+3} = \lim_{x \to -\infty} \frac{6x^2}{7x} = \lim_{x \to -\infty} \frac{6}{7} x = -\infty.$$

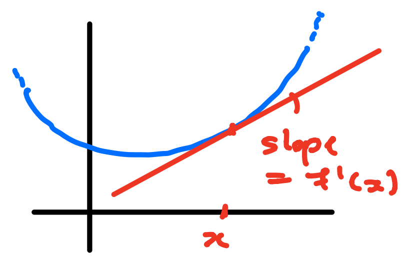
Only works for $x \to \pm\infty$.

Derivatives

derivative of f evaluated at x

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \text{Slope of tangent line to graph at } (x, f(x)).$$

$$= \text{Instantaneous rate of change at } x$$



f differentiable at $x \iff f'(x)$ exists.

Example $\quad f(x) = \dfrac{1}{x} \quad (x \neq 0)$

$$\Rightarrow \quad f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

General Advice : If $f(x)$ looks complicated try simplifying it first.